

Preliminary and Incomplete
Comments Welcome

AN ALMOST STRUCTURALLY EXACT
OVERLAPPING GENERATIONS MODEL
OF THE RENDILLE OF NORTHERN KENYA

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ABSTRACT

This paper makes four contributions to the applied use of overlapping generations (OLG) models. First, it models the social rules of a particular *age-group society*, the Rendille tribe of Northern Kenya, and show that the demographic rules imply an almost structurally exact standard OLG model. Second, despite heterogeneity in the timing of marriage and birthing, the model can be calibrated using standard aggregate demographic data. Third, the calibrated model not only captures population dynamics but also tracks lineages. Fourth, the model permits an analysis of institutions that reveals the intergenerational conflicts in supporting and changing the social rules.

1. Introduction

Anthropologists have identified a number of societies, in various parts of the world, where social and economic life are regulated closely by synchronized transitions through the various stages of life. The rules governing these transitions are well established, and frequently closely linked to chronological age and/or age relative to that of the person's father. Anthropologists call societies with these homogenous lifecycle characteristics *age-set* or, more generally, *age-group societies*.¹ The definitive integrative work in this area is Stewart's *Fundamentals of Age-Group Systems* (1977).

In a recent paper, Engineer and Welling (2004) show that a *standard overlapping generations system* can exactly represent the transitions of Stewart's *standard graded age-set system*, the simplest type of age-group system. The overlapping generations (OLG) structure captures both the generation and the lifecycle stages of individuals.² The generational dimension captures the age-set affiliation; while, the lifecycle dimension captures the grades (social roles) of males by age. Where Engineer and Welling (2004) provide equivalence results for demographic systems, they do not develop a dynamic model of an age-group society.

This paper develops a dynamic model of a particular age-group society, the Rendille tribe of Northern Kenya. The analysis is of interest to economists because we show that the age-group rules followed by the Rendille imply an almost structurally exact standard OLG model. Paying careful attention to the evidence, the rules are modeled in three parts. First, the paternal age-group rules are shown to imply what Engineer and Welling (2004) term a standard OLG system. Second, the marriage rules are shown to incorporate women by age and lineage into the OLG system. Third, fertility is modeled. From these foundations, the OLG model is derived. The model is exact in the sense that

¹ Spencer (1997) describes the overarching premise of such societies as the respect for age. This contrasts with premises such as honor, associated with integrity of kinship, and purity, linked to status and caste.

² The OLG model specifies a well-defined temporal sequence of interactions between agents. Many economists view the OLG model as the paradigm general equilibrium model since it incorporates incomplete markets and lifecycle analysis. See Engineer and Welling (2004) for references to the literature.

both men and women's transitions through their life cycles is synchronized with the beginning of periods in the model.

The OLG model implies restrictions on standard demographic variables and we are able to calibrate it using limited aggregate data on marriage timing and net reproductive rates. The simulations capture population proportions, track lineages, and permit the historical analysis of institutions. A particular marriage institution termed *Sepaade* is shown to dramatically affect the demographics in favor of one lineage. A simple voting analysis reveals the different interests of the generations in the recent decision to eliminate *Sepaade*.

An outline of the Rendille social rules quickly reveals the stylized OLG structure of the demography. In Rendille society, males are assigned by the time of birth to an *age set*. Age sets span 14 years and we model individuals as potentially 6-period lived (where each period is the duration of an age-set). At the beginning of the third period of their lives (between ages 14-28), males are initiated into the *warriorhood* grade. At the end of this period, all warriors marry *en masse* and then become *elders* and can have children. (The lifecycle of father's and sons is illustrated in Figure 1 in Section 2.)

Men typically marry younger women. These *women-marrying-young* marry at the end of their second period of life. They give birth to *early-born children* in the third period of their lives and *late-born children* in their fourth period of life. A peculiar feature of the Rendille age-group system is that daughters of every third age set have a special designation as *Sepaade*. By convention, the marriage of early-born *Sepaade* is delayed by one period. These *women-marrying-old* only give birth to children in the fourth period of their lives. (The timing of marriage is shown in Figure 2 in Section 2.)

The fact that *Sepaade* have lower fertility has been much commented on in the anthropology literature as an example of an institution for population regulation. However, there has been no attempt to model the impact of this institution. The OLG model captures the heterogeneity in the timing of birthing and marriage and reveals that

Sepaade dramatically reduces the level and the growth rate of the population. The model also reveals that *Sepaade* dramatically favors one of the lineages, the *Teeria*, consistent with evidence that this is the most populous and well-off lineage.

A fascinating aspect of Rendille society is their paternal age-group rules governing lineage. Among the Rendille, *age-set lines* represent a well-ordered genealogical lineage that relate father's age set and the eldest son's age set to the same age-set line. The fact that three age sets separate the birth of fathers and eldest sons, indicates that there are three age-set lines and the length of a *generation group* is 42 years.³ The *Teeria*, or "first born", is the age-set line (line X) that starts a *fahan*, a new generation group. The model reveals how the interaction of lineage and *Sepaade* rules act to benefit the *Teeria* age-set line. (The OLG progression of age sets and age-set lines is illustrated in Figure 3 in Section 3.)

The model is used to examine the political economy of coalitions of generations and lineages. We start with a counterfactual analysis investigating the origin, persistence and recent dissolution of the institution of *Sepaade*. Our analysis is consistent with Rendille oral history that *Sepaade* arose in 1825 as a reaction to external threat: it mobilized women's labor for camel herding while the men engaged neighboring tribes in battle. We speculate that *Sepaade* persisted because it favoured the *Teeria* age-set line and the dominant position of the *Teeria* allowed them to block attempts to reverse it. *Sepaade* was recently discontinued in 1998. Our analysis suggests this might happen for two reasons. First, the society was becoming too demographically lopsided towards the *Teeria* line to the substantial detriment of the overall fitness of the society. Second, the institution became unviable as women disadvantaged by the tradition found it easier to emigrate in recent times.

We are aware of no direct precedents for our analysis. In the economics literature, only recently have economist been able to operationalize the OLG model due to

computational complexity and data limitations. The few attempts have concentrated on modeling macro variables like the aggregate capital stock rather than accurately capturing the heterogeneous transitions of agents through life cycle stages by modeling marriage, fertility and tracking lineages.⁴ In the anthropology literature, the closest analysis is probably Legesse's (1972) macro simulation analysis revealing the internal inconsistency of the age-group rules of the Borana. The "experimental history" literature in demography (e.g. Hemmel (1979)) also adopts a similar counterfactual approach to understanding social rules but concentrates on identifying statistical variations in micro simulations. The related literature outside of economics is reviewed in Engineer, Roth and Welling (2004). This paper also provides a detailed historical background of the Rendille and documents the simulation model used in the current paper. Variants of the simulation program are used to explore the history more completely.

The paper proceeds as follows. The next section summarizes the social rules of the Rendille age-group society. Section 3 models the social rules. Section 4 develops the overlapping generations model and analytically describes the model's dynamics. The model is calibrated in Section 5. Section 6 develops a simple voting analysis of the decision to abolish *Sepaade*. Section 7 concludes.

2. The Age-Group System of the Rendille

The Rendille are a Cushitic-speaking people with a population of approximately 30,000, who live in Northern Kenya. In the northern Kenyan lowlands the land is too dry for farming; nomadic pastoralism is the most efficient - and possibly the only - way to sustain life in this environment. The Rendille have camel herds, with some additional smallstock. Wealth among the Rendille is measured by the camel herds, owned by the (male) elders in the tribe. These herds are passed on to the eldest son in a family. The

³ The word "generation" is often confused in two uses -- as a measure of time associated with a cohort of individuals versus genealogical distance between parent and child. In the OLG literature, the term "generation" most often corresponds to an age set versus a generation group.

⁴ Applications of the OLG model that include considerable demographic detail include Auerback and Kotlikoff (1987), Gokhale et. al. (2001), and the related work on generational accounting and social

Rendille rely upon camels as a tradeable good, as a means for transporting their communities and water, and as a source of food (milk most importantly, blood, and occasionally meat). Until recent times, interaction with agricultural and industrial economies was limited to the trade of skins or livestock itself for tea, sugar, maize, tobacco, cloth, etc. (Beaman p. 29). For more complete description of Rendille society see Beaman (1981), and Roth [(1993), (1999), (2001)].

Our focus is on Rendille age-group organization which goes back well before 1825 when the institution of *Sepaade* was introduced (Roth (1993), p601). Beaman (1981) provides the most comprehensive analysis of the system. We organize our description by first listing Beaman's rules for the age-group organization and then detail important aspects of the rules relating to the paternal age-group system, marriage, and fertility.

Beaman's (1981, p380-423) lists 15 rules governing the Rendille age-group society. The complete list of these rules and a full discussion are in Engineer et. al. (2005). Here we list the 11 rules relevant to our model in Table 1.

Table 1:

BEAMAN'S RULES ON

RENDILLE AGE-GROUP ORGANIZATION (ABRIDGED)

1. There are three grades: boyhood (from birth to initiation; this may be termed a pre-grade); warriorhood (from initiation to marriage); and elderhood (from marriage until death).
2. A new age-set is formed every fourteen years upon the initiation, by circumcision, of all eligible boys into warriorhood.
3. Only one age-set occupies the warrior grade at a time.
4. Warriorhood confers the right to engage in sexual relations but not the right to claim paternity in any child, which comes only from marriage.

security. Hugget (1996) calibrates an OLG model to describe the U.S. wealth distribution. Greenwood et. al. (2003) calibrate an OLG model that includes marriage and fertility to the wealth distribution.

5. Warriors remain unmarried for eleven years after initiation, and then all members of set become eligible for marriage and elderhood at one time.
6. A son is normally circumcised into the third age-set to follow that of his father; late-born sons may join later sets, but no son may be circumcised with an earlier set.
7. A late born son, too young for circumcision with third age-set after his father's, may by special prearrangement "climb" into it after circumcision with the succeeding set. ...
8. The age-sets are organized into three lines of descent as a result of Rule No. 6, such that every third set is composed largely of sons of the first. Fathers and sons thus tend to fall into the same age-set lines. One set in each line is inaugurated before any line recurs.
9. One age-set line is named *Teeria*, and is considered the senior line of the three. The sons of any *Teeria* man, if they are initiated into the third set after their father's, will be *Teeria* themselves.
10. Daughters of *Teeria* men are called *Sebade* (or *Sepaade*), and in most lineages they are not allowed to marry until their brothers have become eligible to do so.
11. No son born to a woman after her eldest son has been circumcised may be raised, and such a son should be killed at birth.

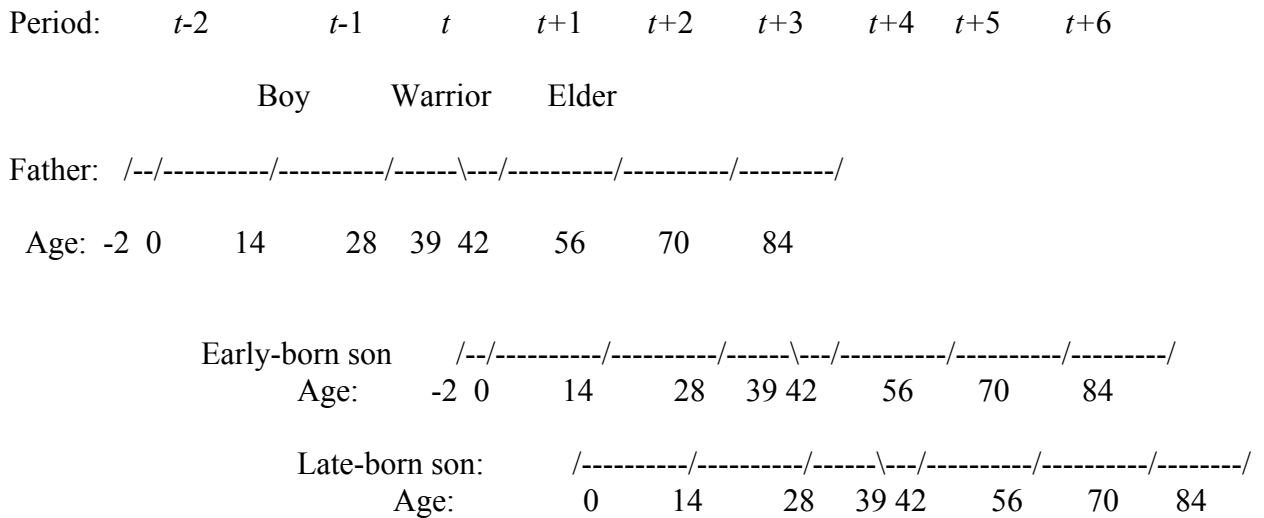
2.1 *The Paternal Age-Group System*

Rules 1-9 describe the lifecycle of men, and the assignment of men and their sons to age groups or "age sets". The circumcision of a group of boys marks the beginning of a new age-set and the transition from boyhood to warriorhood. The group of warriors initiated in period t is then identified as age-set t . The next circumcision occurs 14 years later, at which point age-set $t+1$ forms. After 11 years as warriors, men become eligible for marriage and elderhood (following the *nabo* ceremony). After 3 years of readjustment and adaptation to new roles, the next age-set is opened; thus, during the 3-year gap before the next initiation there are technically no warriors.

The timelines for a father and his son are described in Figure 1, using 14-year periods. The father is of age-set t , the period during which he is a warrior. Suppose he is

born at the beginning of period $t-2$. If he lives to the end of period $t+2$ (or beginning of period $t+3$), he would be 70 years old when he dies. After two periods in boyhood, this male enters warriorhood at age 28 and marries when he is aged 39 to 42.⁵ Rule 4 requires that males can only claim paternity of children from marriage.

Figure 1
Father and sons



Now consider sons. Stewart (p 108) states: “Fundamentally, the Rendille operate a *negative paternal-linking system*.” A negative paternal-linking system restricts the minimum number of age sets separating the inauguration of father and son into their respective age sets and organizes fathers’ and early-born sons’ age sets into age-set lines. Beaman’s rules 6 and 8 sketch the operation of this system among the Rendille.

Rule 6 restricts the minimum age distance between fathers’ and “early-born” (i.e. eldest) sons’ inaugurations to be three age sets or 42 years. Thus if the father is inaugurated into age-set t the early-born sons are inaugurated into age-set $t+3$. By rule 8, every third age set belongs to the same *age-set line* so that age-set t and $t+3$ belong to the

⁵ An age-set t male born at the end of the period $t-2$ enters warriorhood at age 14 and marries when he is 25-28. If he lives to the end of period $t+3$ he would be 70 years old.

same age-set line. Age-set lines preserve the generation-group relationship between fathers and the eldest sons (who receive all the family wealth, primogeniture). Late-born (i.e. younger) sons join later age sets and change their age-set line accordingly. This is called a negative paternal-linking system because of these features.

Figure 1 depicts “early-born” sons as born in the interval starting one year after the period t *nabo* ceremony and extending through period $t+1$. Typically most sons are born in this interval. It implies that sons of between ages 14-30 at the beginning of period $t+3$ are initiated into age-set $t+3$. Beaman (p.388) states: “As the majority of sons are not first born, the majority of initiates at circumcision are younger than thirty.” Except for first-born sons (born at the end of period t), the figure uses an enrolment age of 14.⁶ Thus, sons born in period $t+2$, late-born sons, are too young to be circumcised at $t+3$ and are circumcised at same times as their “age mates” at the beginning of $t+4$.⁷

Rule 8 specifies that every age set is assigned to one of three age-set lines in rotation. For example, consider the *Teeria* line, identified in Rule 9 as the senior age-set line. If age-set 0 is in the *Teeria* line then so are age sets 3, 6, 9, As early-born sons are normally enrolled in the third age set following their fathers', they are in the same line as their fathers. Late-born sons are usually enrolled in the subsequent line.

The Rendille system deviates from the norm of negative paternal linking by a unique institution called *climbing*. According to Rule 7, by special prearrangement, a late-born son of an age-set t father may “climb” into age set $t+3$.⁸ Climbing involves

⁶ Beaman (p389) states: “...boys younger than fourteen are seldom among the initiates”. Stewart (1977) cites Baxter as observing a son of about 12 being initiated. Adamson (1967) interviews with the Rendille indicated no sons younger than 14 being initiated. Our discussions with Rendille elders confirm that (except for first-born sons) one age-set is the usual minimum interval between the birth and inauguration of a son. This historical practice has been violated recently with younger sons being circumcised.

⁷ Sons sired by age-set t fathers in period $t+3$ are initiated into age-set $t+5$. However, the number of such sons is likely to be small for two reasons. First, Rule 15 requires that no women with sons circumcised in period $t+3$ shall raise sons in that period or later. Second, period $t+3$ corresponds to husbands being between 56-84 years old. It is likely that many of these husbands are already dead or too old to sire.

⁸ If the late-born son is considered too young to climb then the right can be transferred to the grandson and so on. Beaman’s rule 7 states: “As the right to “climb” if not used by the eligible son, is passed down to his own sons.” Stewart (1977, p.111) and Roth (1993, p.602) describe climbing as a right that in principle is extended in perpetuity.

circumcision with one's age mates, then proceeding immediately to elderhood (skipping warriorhood), with the right to marry without delay. Elders report that historically only one or two persons climb in any clan, which would put the proportion that climb below 5%.⁹ Climbing is relatively rare because it usually occurs when a father has only late-born sons and tries to preserve at least one son in the family's historical age-set line.¹⁰ In Section 3 we chose not to model climbing partly because climbing is infrequent and also because we argue that it does not essentially affect the long-run population trends.

2.2 Marriage

Traditionally, all men are strongly encouraged to marry as soon as possible, and most men of a given age-set marry in a mass ceremony shortly after their *nabo* ceremony. Marriage involves the husband paying bride wealth of 4 cattle to the wife's family. This is a substantial payment that the eldest son usually acquires from his father (or inheritance) by the time of the *nabo* ceremony. The Rendille subscribe to primogeniture so that by custom only the oldest son receives the family wealth, livestock.

Younger sons, who work for their father or older brother during warriorhood in the *fora*, have a moral right to, but are not assured of, cattle for bridewealth. This often gives rise to conflicts between brothers. Poor men who cannot raise bride wealth cannot marry right after the *nabo* ceremony but instead do 2-3 years bride service for their prospective in-laws before marrying. Almost all poor men are usually married by the end of the three-year transition period to elderhood. Alternatively, faced with the prospect of

⁹ Elders views on climbing were gathered by Engineer in interviewed in May 2001. Beaman's (1981) census of 136 men finds that only 2.4% of men are initiated early, i.e. two age sets after their father's. In a smaller census Spencer as reported in Stewart (1977) found around 10% of men climb.

¹⁰ The fact that climbing is uncommon is supported by additional considerations. First, the main reason put forward for climbing is to secure an heir in the father's line. However, primogeniture results in the wealth being inherited by the oldest son. Thus, the presence of early-born sons precludes the need for late-born sons to climb. A late-born son will climb only when there are no sons born in the first 17 years following the *nabo* ceremony. Second, climbing is not in the economic interest of the father: it precludes the son from warriorhood and hence from working for the father in the *fora*. Also, it puts the onus on the father to give up cattle early so the son can pay bride wealth. Lastly, elders report in interviews (with Engineer in 2001) that very young sons do not want to climb but would rather be in their age mates' age set.

not receiving cattle, younger sons often emigrate before or during warriorhood to the neighbouring Samburu where doing bride service is less onerous.

The Rendille permit polygyny. However, Spencer (1973) reports that when he observed the Rendille in the early 1970s, the vast majority of marriages were monogamous. If a man took a second wife it would typically be years after the first marriage, in a separate ceremony. Roth (1993) states that the chief reason for taking a second wife was to have a son if there was no male issue from the first marriage.

Figure 2
Husbands and Wives

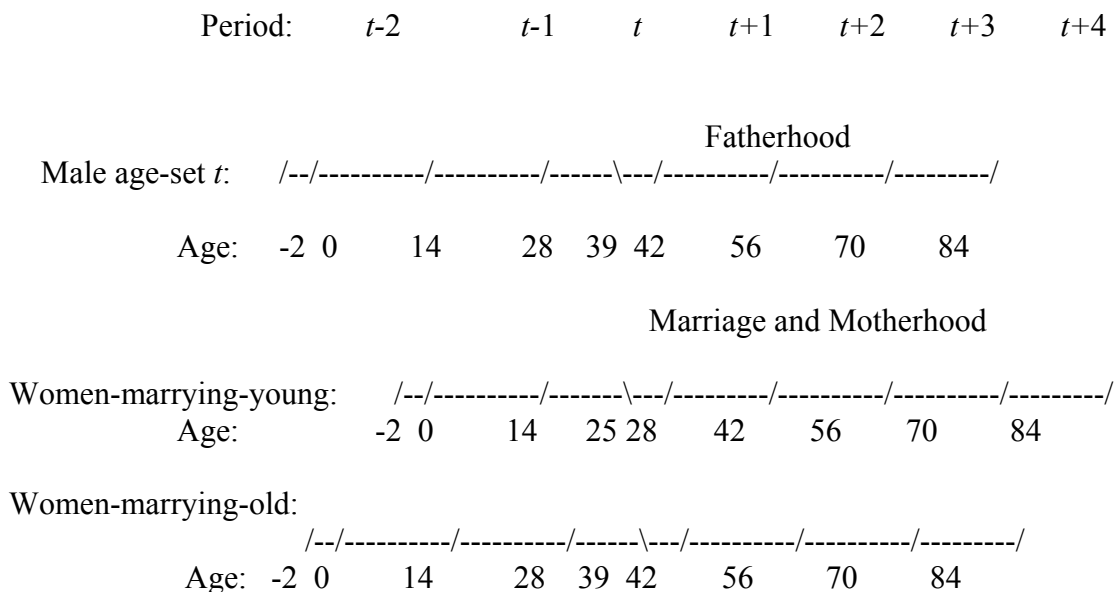


Figure 2 illustrates the traditional timing of marriage for males of age-set t . These men marry after the *nabo* ceremony at the end of period t . In the absence of *Sepaade*, the usual practice is for men to marry younger women, sisters of men one age-set their junior. We denote these women as *women-marrying-young*. In the figure, such a female born at the beginning of period $t-1$ would be 25 when she marries. Females born in the last two years of period $t-1$ would be 11-13 at the time of the next *nabo* ceremony. Since girls are typically not eligible for marriage until age 14, such girls usually marry a few years later to men doing bride service (or wealthy men taking second wives). First-born

daughters of age-set $t-1$ fathers are an exception to this rule; they are usually forced to delay their marriage to the following *nabo* ceremony. Similarly, first-born females of age-set $t-2$ fathers who are born in the last two years of age-set $t-2$ marry age-set t men, in which case they may be as old as 27 when they marry.¹¹

Women whose marriage is delayed by a full *nabo* ceremony we term *women-marrying-old*. In the figure, women-marrying-old to age-set t men are born in period $t-3$. Thus, women-marrying-old marry into the age-set of their same-aged brothers and will be between ages 28-44 at the time of the *nabo* ceremony. Most women-marrying-old do so because they are held back from marrying by the institution of *Sepaade*.

Rule 10 indicates that all daughters of *Teeria* men (age-set line X) are designated *Sepaade*. The social rules on *Sepaade* delay marriage and in its place assign special work. Typically, the rules delay the marriage of early-born *Sepaade* daughters by one age-set so that they become women-marrying-old as described above (see Figure 2).¹² *Sepaade* women delayed in marriage do the same work as sons, herding camels for their fathers.¹³ For the work they do and the fact that they marry back into the *Teeria* lineage, *Sepaade* are considered by the Rendille as the main reason why the *Teeria* are more numerous and wealthy than the other age-set lines.¹⁴

2.3 *Reproduction*

The delays in marriage caused by *Sepaade* clearly reduce their fertility. Beaman explicitly lists one other rule related to fertility: “No son born to a woman after her eldest

¹¹ There are two reasons to believe this is the case. First, first-born daughters are valuable to the families for the work they do. Second, our data contains surprisingly few married young women in the range of 14-18. Thus, it is unlikely that they marry in the transition period following the *nabo* ceremony to age-set $t-1$ men.

¹² Anthropologists accounts suggest this timing. An exception is Roth (2001, p1017) who interprets the *Sepaade* rule as potentially further delaying marriage for a small fraction of daughters. As shown in Engineer et. al. (2004) this leads to similar results with slightly lower growth rates.

¹³ This tradition is also interesting because it arguably defines a third gender where *Sepaade* perform a hybrid role. Beaman (1981, p.435) believes that *Sepaade* is an ecologically adaptive control on population growth in a restrictive environment. This raises the intriguing possibility of viewing *Sepaade* as a gender response to ecological adaptive control.

son has been circumcised may be raised, and such a son should be killed at birth.” Under this rule, a women-marrying-young in period t and bearing an early-born son in period $t+1$ can not have a very late-born son in period $t+3$.

Beaman mentions several additional important restrictions on fertility that she does not list as rules. First, unmarried women are forbidden to bear children. Second, polygyny is allowed. This implies the shortage of men will not affect the timing of marriage. Third, widows cannot remarry but can continue to bear children (Beaman p. 104). Fourth, any children born to widows are assigned as if they were the husband's. Thus the husband's death does not interrupt the usual passing down of lineage and does not restrict fertility. These rules and restrictions all indicate that only the females age at time of marriage matters for lifetime fertility.

3. Modeling the Age-Group Rules as an OLG System

This section first shows that the Rendille paternal age-group rules imply a *standard overlapping generations system*. Second, the marriage rules are shown to incorporate women by age and lineage into the OLG system. Third, fertility is modeled as depending only on female age at time of marriage. From these foundations, the “maternal” OLG model is derived in Section 4.

Before consider specific rules, we first describe the time grid for the demographic system. Almost all Rendille demographic activities can be naturally and parsimoniously represented relative to the initiation of age sets as described in Figures 1 and 2.¹⁵

Assumption 1 (Time and Period Length). Time is partitioned into discrete 14-year periods. The period begins with the initiation of an age set and ends with the initiation of the subsequent age set. Periods are indexed by whole numbers.

¹⁴ Interviews with Rendille elders in 2001 by Engineer. *Sepaade* are considered to be very valuable to father's household.

This assumption allows us to trace the life passages of cohorts by their period of birth and their age in periods. However, there is one obvious feature that is “out of joint” with the period increment. Not all early-born children are born in the same period. Early-born children of men age-set t in Figure 1 are born over a 16-year interval that straddles the last two years of period t and extends for all of period $t+1$. The children born in the last two years of period t are *first-born* children. These first-born children do not pose a problem for the analysis because, as discussed in Section 2, the social rules treat all early-born children the same (the age-set designation and marriage timing is determined by their father’s age set). Thus, there is no loss of generality in technically lumping the first-born children in the period following the marriage of their parents.

Simplifying Assumption (Early-born children). All early-born children are born in the period immediately following the marriage of their parents.

For the calibration this assumption works out nicely as our data is by lineage.

3.1 *Modeling the Paternal Age-Group System*

With the Simplifying Assumption, Rendille paternal demography can be almost exactly modeled using 14-year periods.

Assumption 2 (Individuals and Age Sets). The lifespan of an individual potentially spans six contiguous 14-year periods. Males born in period $t-2$ are inaugurated into *age-set* t at the beginning of period t ; this period defines the age-set number for that cohort.¹⁶

¹⁵ The Rendille calendar is in lunar years and is not completely synchronized with the Julian calendar. However, the period length is roughly the same in both calendars as described by Beaman (1977). Roth (1993) traces Rendille initiations back to 1853 using 14-year periods in the Julian calendar.

¹⁶ In Assumption 2 age-set assignment is by age whereas it is by paternity in Beaman’s Rule 6. The two coincide under the simplifying assumption for early-born children. Using Rule 6 makes no practical difference to the simulations.

The role of males is determined by their age-grade assignment.

Assumption 3 (Age-Grades). Age grades are assigned to males in a way that coincides with periods. The period of his birth and the following period are *Boyhood*. The third period of life is *Warriorhood*. The remaining periods of life correspond to *Elderhood*.

The lifecycles of males is illustrated in Figure 3. Consider age-set j males. These males are boys in periods $j-1$ and $j-2$, warriors in period j , and elders in periods $j+1, j+2$, and $j+3$. The number of boys is denoted $B_1(j-2)$ and $B_2(j-1)$, where the subscript indicates their age in periods. Similarly, warriors are denoted $Wr_3(j)$, and elders are $E_4(j+1)$, $E_5(j+2)$, and $E_6(j+3)$. Since births of individuals (who are recruited into any cohort) occur throughout the period, the variables count people at the end of the period. This captures the extant population in the age-set.

Figure 3
Paternal Graded Age-Set System

		Time									
		$j-2$	$j-1$	j	$j+1$	$j+2$	$j+3$	$j+4$	$j+5$	$j+6$	$j+7$
Line	Age Set										
X	j	B_1	B_2	Wr_3	E_4	E_5	E_6				
Y	$j+1$		B_1	B_2	Wr_3	E_4	E_5	E_6			
Z	$j+2$			B_1	B_2	Wr_3	E_4	E_5	E_6		
X	$j+3$				B_1	B_2	Wr_3	E_4	E_5	E_6	
Y	$j+4$					B_1	B_2	Wr_3	E_4	E_5	E_6
Z	$j+5$						B_1	B_2	Wr_3	E_4	E_5

Figure 3 is typical of depictions in the OLG literature and exactly corresponds to what Engender and Welling (2004) define as a *standard OLG system*.¹⁷ The system describes both the generation and the lifecycle stages of individuals. The generational dimension captures the age-set affiliation; while, the lifecycle dimension captures the

¹⁷ The definition includes six elements: time, agents and generations, endpoints, period length, lifecycle stages, and stationarity. Stationarity refers to the unique mapping from age to lifecycle stage that is independent of the time period. Figure 3 satisfies all of these elements for a “perpetual economy” with no endpoints. The proof is straightforward but tedious and therefore is omitted.

grades (social roles) of males by age.¹⁸ The figure illustrates the complete cross-section of males in different grades for period $j+3$.

Figure 3 depicts one additional feature not found in economic depictions. Every third Rendille age set belongs to the same age-set line.

Assumption 4 (Age-Set Lines). The three Rendille age-set lines follow a cycle as follows: *age-set line X* includes age-sets $j = 0, 3, 6, 9, 12, 15\dots$; *age-set line Y* includes age-sets $j+1 = 1, 4, 7, 10, 13\dots$; and *age-set line Z* includes age-sets $j+2 = 2, 5, 8, 11, 14, \dots$. Age-set line *X* is the *Teeria* age-set line.

Age-set lines are part of the Rendille *negative paternal-linking system*, which preserves lineage from fathers to their early-born sons. According to Assumption 2, sons born in period $t+1$ are initiated into age-set $t+3$, whereas sons born in $t+2$ are initiated into age-set $t+4$. If these are sons of fathers t , then they are respectively early-born sons age-set $t+3$ and late-born sons age-set $t+4$. The early-born sons are in the father's line, whereas the late-born sons fall into the subsequent line. In Figure 3, fathers j are in line X and so are their early-born sons, age-set $j+3$. Late-born sons $j+4$ fall into line Y.¹⁹

The mapping from Beaman's rules to the above assumptions is largely obvious. The mapping is inexact in Assumption 2 where age-set assignment is by age whereas it is by paternity in Beaman's rules 6 and 7. The two coincide under the Simplifying Assumption for early-born children when there is no "climbing". Climbing violates Assumptions 2 and 3; climbers are typically late-born sons who do not join the age-set of their age mates but by special prearrangement skip warriorhood and join the senior age-set. This puts them in the age-set line of their fathers. Our analysis ignores climbing

¹⁸ Conversely, Assumptions 2 and 3 satisfy what Stewart defines as a *graded age-group system*. In fact, because initiations are at the beginning of periods and all periods are of equal length, it describes a particularly well-behaved system that Engineer and Welling (2004) term a *standard graded age-set system*. Engineer and Welling (2004) show that this system is equivalent to a *standard OLG system*.

¹⁹ Stewart (1977, 104) notes that negative paternal linking does not preclude the society from adhering exactly to the *age-set model*. Engineer and Welling (2004) show that the age-set model is consistent with the *standard OLG system*.

because, as detailed in Section 2.1, climbing only applies to a small proportion of sons so, as noted by Stewart (1977: 108), “Fundamentally, the Rendille operate a negative paternal-linking system”. In our maternal model, the impact of a male climbing in the current period does not affect the future population. This is because climbers sire children at the same time as others in their adopted age set. Climbing only has minor effects on the proportion of males in the lines.²⁰

In addition to climbing, there is one other aspect in which our standard OLG system depiction of the Rendille paternal age-group rules is inexact. In Assumption 3 warriorhood occupies the entire 3rd period of life; whereas in actuality, the transition between warriorhood and elderhood is three years before the opening of the next age set. However, our simplification is arguably inconsequential as males in transition are the effective warriors and are putative elders. We refer to the standard OLG system as an “almost exact” depiction of the paternal age-group rules because it accurately describes the initiation of the almost all males and their transit through the Rendille grade system. The exceptions are not important for modeling the age-set demographics.

3.2. *Modeling Marriage*

In Rendille society failure to marry is greatly frowned upon and *polygyny* (multiple wives) is allowed. All women marrying is feasible as polygyny implies there need not be a shortage of husbands. As the usual pattern is for a man to have either one or two wives, we make the following assumption.

Assumption 5 (Marriage, and Polygyny) All women marry. All men marry (if possible), and no man has more than one more wife than any other man.

The specification that “no man has more than one more wife than any other man” allows us to derive simply the distribution of polygynous marriages. For example, if the

²⁰ Climbing involves complicated tracking of sons and fathers and grandfathers and is modeled in our simulation program. In the case when the climbing rates are the same across age-set lines (we have no

polygyny ratio R of married women to men is $R=1.1$, 10% of males will have 2 wives and 90% will have 1 wife. The polygyny ratio exceeds 1 with population growth or child rearing practices that favour girls over boys.

Males of age-set t , $Wr_3(t)$, marry after the *nabo* ceremony near the end of period t . Under the Simplifying Assumption, they marry females who are either *women-marrying-young* born in period $t-1$, or *women-marrying-old* born in period $t-2$.

Assumption 6 (Marriage Timing) All men of age-set t marry at the end of period t (or equivalently, at the beginning of period $t+1$). They marry to *women-marrying-young* born in period $t-1$ and to *women-marrying-old* born in period $t-2$. These are the only groups of women from whom they draw marriage partners.

The factors that determine whether a female marries young or old are the lineage of her father and whether she is early- or late- born. In particular, *Sepaade* restricts most early-born daughters of Teeria men to be *women-marrying-old*. The following definition develops notation for modeling the timing of marriage.

Definition. Let p_X , p_Y and p_Z denote the proportions of *early-born* daughters in lines X, Y, and Z respectively that are *women-marrying-young*. Similarly, let p_X' , p_Y' and p_Z' denote the proportions of *late-born* daughters in lines X, Y, and Z respectively that are *women-marrying-young*.

Sepaade implies that p_X is small or, equivalently, that $1-p_X$ is large. That is, most early-born daughters in line X are *women-marrying-old*. Complete *Sepaade* is $p_X=0$. The following assumption specifies conditions capturing the relative import of the restrictions on the various cohorts.

Assumption 7 (Marriage Proportions by Line) *Sepaade* restricts most early-born daughters of Teeria men to be *women-marrying-old*, $p_X \leq 0.4$. The vast majority of early-

evidence to the contrary), the proportions of men in the lines is only slightly altered.

born daughters in other lines are women-marrying-young, $p_Y = p_Z = p \geq 0.8$. Almost all late-born daughters are women-marrying-young $p_X' = p_Y' = p_Z' = p' \geq p$.

We state the assumption this way to give a sense for the magnitudes consistent with our limited evidence (presented in Section 5). Our analysis of *Sepaade* in this paper maintains $p_X < p \leq p'$. As we have no evidence to the contrary, we equate the proportions in the other lines: $p_Y = p_Z = p$; also $p_X' = p_Y' = p_Z' = p'$. Thus, the analysis isolates *Sepaade* as the key asymmetry.

Assumptions 6 and 7 allows us to identify the specific groups of women who marry men of age-set t , $Wr_3(t)$, and to identify these women as daughters of earlier age sets. To do this we introduce some notation. Girlhood in the period of birth is denoted G_1 , if the female is early born and G_1' if she is late born. The lifecycle of females depends on the timing of their marriage. Females who marry at the end of the second period of their lives are called women-marrying-young. They are denoted WMY_2 if they are early born, and WMY_2' if they are late born. Females that are women-marrying-old go through another grade denoted G_2 or G_2' in the second period of their lives before marrying. They marry at the end of the third period of their lives and are denoted WMO_3 if early born and WMO_3' if late born.

Thus, there are potentially four groups of women that marry men $Wr_3(t)$ in period t : early-born and late-born women-marrying-young, $WMY_2(t)$ and $WMY_2'(t)$, and early-born and late-born women-marrying-old, $WMO_3(t)$ and $WMO_3'(t)$. Recall from Assumption 6 that women-marrying-young in period t are born at $t-1$ and that women-marrying-old in period t are born at $t-2$. Thus, we can track the women groups back through their lifecycle to the new-born girl groups from which they are drawn. This is done in Table 2, ignoring attrition. Consistent with Figure 3, men marrying in period j , $Wr_3(j)$, are from line X.

Table 2
Marriages of Daughters by Line to Men by Line

Line, Men	Daughters of X (<i>Teeria</i>)	Daughters of Y	Daughters of Z
X (<i>Teeria</i>), $Wr_3(j)$	$WMO_3(j)=(1-p_X)G_1(j-2)$ $WMY_2'(j)=p_X'G_1'(j-1)$	$WMY_2(j)=p_YG_1(j-1)$	$WMO_3'(j)=(1-p_Z')G_1'(j-1)$
Y, $Wr_3(j+1)$	$WMO_3'(j+1)=$ $(1-p_X')G_1'(j-1)$	$WMO_3(j+1)=(1-p_Y)G_1(j-1)$ $WMY_2'(j+1)=p_Y'G_1'(j)$	$WMY_2(j+1)=p_ZG_1(j)$
Z, $Wr_3(j+2)$	$WMY_2(j+2)=p_XG_1(j+1)$	$WMO_3'(j+2)=(1-p_Y')G_1'(j)$	$WMO_3(j+2)=(1-p_Z)G_1(j)$ $WMY_2'(j+2)=p_Z'G_1'(j+1)$

Table 2 extends the OLG system to organize all females by period of marriage and traces them back to their father's line according to the proportion that marry young and old. A sense for how the proportions impact the dynamics is revealed by the polygyny ratio. The polygyny ratio measures the average number of wives per man. For age-set j men from line X, illustrated in Table 2, the polygyny ratio is:

$$R(j) = [WMO_3(j) + WMY_2(j) + WMY_2'(j) + WMO_3'(j)] / Wr_3(j). \quad (1)$$

$$= [(1-p_X)G_1(j-2) + p_YG_1(j-1) + p_X'G_1'(j-1) + (1-p_Z')G_1'(j-2)] / Wr_3(j).$$

Sepaade benefits line X men -- a reduction in p_X increases the number of line X girls $G_1(j-2)$ marrying into line X at the expense of line Z. Consider an extreme case with "Complete *Sepaade*", where $p_X = 0 < p = p' = 1$. Then the number of women that line X men $Wr_3(j)$ marry is $WMO_3(j) + WMY_2(j) + WMY_2'(j) = G_1(j-2) + G_1(j-1) + G_1'(j-1)$, whereas line Z men $Wr_3(j-1)$ marry only $WMY_2'(j-2) = G_1'(j-2)$ women. This *Sepaade* induced asymmetry shows up in the difference equations derived below and can result in line Z dying out.

3.3 Modeling Reproductive Rates

The following assumption specifies female net reproductive rates, i.e. surviving daughters per mother.²¹

Assumption 8 (Net Reproductive Rates). Women-marrying-young each rear n_1^y daughters born in the first period after marriage, and rear n_2^y daughters born in the second period after marriage, where $n_1^y \geq n_2^y > 0$. Women-marrying-old each rear n^o daughters born in the first period after marriage. Lifetime net reproductive rates are restricted:

$n^y \equiv n_1^y + n_2^y \geq n^o$ and $n^o \geq n_2^y > 0$. Of the children reared, the gender ratio of males per female is g . No children are reared before marriage.

The lifetime net reproductive rate for women-marrying-young is generally greater (and never less) than for women-marrying-old, $n^y > n^o$. This is because women-marrying-young bear children for two periods (average of 28 years) following marriage whereas women-marrying-old only bear for one period (average of 14 years) following marriage. The period net reproductive rates are declining with the number of periods following marriage, $n_1^y > n_2^y$. These fecundity rates and intervals roughly match Roth's (1993, 1999) data.

The reproductive rates are conditional on the female's age at time of marriage and not on a host of other considerations. This type of female-based fecundity assumption is often made in demographic research to simplify the analysis. We believe that if there is any society that fits this assumption the Rendille are an excellent candidate. Recall from Section 2.3, in Rendille society child rearing is only permitted after marriage. All children born of a married woman are brought up as if they were the husband's, even well after the husband is deceased. Rearing rates are independent of the presence of the husband or number of other wives. Finally, reproductive rates are independent of whether mothers were early-born or late-born.²²

²¹ The simulation program allows for gross reproductive rates and a schedule of mortality rates.

²² Even if these considerations were important (and we have no evidence that they are), it is not clear they would affect the population proportions, unless they impacted the lines asymmetrically.

4. The OLG Model and Dynamics

4.1 The OLG Model

The OLG system described in Table 2 can be combined with the net reproductive rates to derive the dynamic equations of the OLG model. The equations relate mothers to daughters through time. Given initial conditions for females, the equations yield a dynamic path for the maternal demography. The paternal demography can be easily derived from this solution.

Consider all daughters born in period $t+1$. Early-born daughters, $G_1(t+1)$, are daughters of $E_4(t+1)$ men. Suppose these men belong to age-set j and line X. From Table 2, we know that these men in the previous period, $Wr_3(j)$, married four groups of women: women-marrying-young $WMY_2(j)=p_Y G_1(j-1)$ and $WMY_2'(j)=p_X' G_1'(j-1)$, and women-marrying-old $WMO_3(j)=(1-p_X)G_1(j-2)$ and $WMO_3'(j)=(1-p_Z')G_1'(j-1)$. Hence, ignoring attrition early-born daughters born to line X fathers are:

$$G_1(j+1) = n_1^y \{p_Y G_1(j-1) + p_X' G_1'(j-1)\} + n^o \{(1-p_X)G_1(j-2) + (1-p_Z')G_1'(j-2)\}$$

Late-born daughters born in period $j+1$ are from women-marrying-young in period $j-1$ to men from line Z: $WMY_2(j-1)$ and $WMY_2'(j-1)$. Hence, late-born daughters born to line Z men are:

$$G_1'(j+1) = n_2^y \{p_X G_1(j-2) + p_Z' G_1'(j-2)\}.$$

Similarly we can derive the girls born in periods $j+2$ and $j+3$.

$$G_1(j+2) = n_1^y \{p_Z G_1(j) + p_Y' G_1'(j)\} + n^o \{(1-p_Y)G_1(j-1) + (1-p_X')G_1'(j-1)\}$$

$$G_1'(j+2) = n_2^y \{p_Y G_1(j-1) + p_X' G_1'(j-1)\}.$$

$$G_1(j+3) = n_1^y \{p_X G_1(j+1) + p_Z' G_1'(j+1)\} + n^o \{(1-p_Z)G_1(j) + (1-p_Y')G_1'(j)\}$$

$$G_1'(j+3) = n_2^y \{p_Z G_1(j) + p_Y' G_1'(j)\}.$$

This completes the rotation through lines X, Y, and Z, after which the system repeats. The system relates daughters of men in one genealogical generation to daughters of men in the next genealogical generation.

The linear six-equation fifth-order difference equation system is easy to simulate. Our accompanying paper, Engineer et. al. (2004) contains documentation for a simple Excel spreadsheet that instructs the reader on how “experiment” with the model and various historical scenarios. The system can also be solved analytically. Engineer and Kang (2003) prove the existence of a unique globally stable dynamic path that converges to a (periodic) steady-state growth path.²³ They also find necessary and sufficient conditions for growth of the age-set lines. Because the dynamical analysis is technically involved, we only refer to the simpler and more striking results below.

4.2 Symmetry

Symmetry among the age-set lines requires $p = p_X = p_Y = p_Z$ and $p' = p_X' = p_Y' = p_Z'$. “Full Symmetry” is the further requirement that $p = p'$ and $g=1$. The following result describes the growth of all our variables describing population series and aggregates (eg. men in age-set lines and total population).²⁴

²³ The system is formulated in “generation-group periods”, where a rotation through the lines X, Y, and Z constitute one period of 42 years in duration. This reduces the dimensionality of the dynamical system and also turns out to be a more exact characterization of Rendille society.

²⁴ The exceptions are aggregates that fluctuate over duration of a *fahan*. For example, the number of women by age-set line varies with the period. Every third period it includes two groups, one just married and women about to die; whereas, in other period it includes just one group of women.

Results (Full Symmetry): (a) $n^y = n^o = 1$; the model converges to a steady state with zero population growth and $R=1$. The steady state population is increasing in p and in n_1^y (holding $n^y = n_1^y + n_2^y = 1$).

(b) With $n^y > n^o \geq 1$, the model converges to a steady state with population growth and polygyny ratio $R > 1$. The steady state population growth rate and polygyny ratio are increasing in n^y , n_1^o , and p . The population level is increasing in p and in n_1^y holding $n^y = n_1^y + n_2^y$ constant.

With each mother rearing one daughter and equal gender proportions, there is no polygyny and the steady state population is constant. The steady state population level is larger when more children are born early and less late, because the average lag between birth and bearing children is reduced.

More generally, population growth occurs when n^y is sufficiently large.²⁵

Population growth induces polygyny as long as there are some women-marrying-young, $p > 0$. Then the pool of marriageable women is larger than men. From (1) we can derive

$$R(t) = [1/g] * \{(1-p) + p * [G_1(t-1) + G_1'(t-1)] / [G_1(t-2) + G_1'(t-2)]\}.$$

In the steady state the gross growth rate of the population is $[G_1(t-1) + G_1'(t-1)] / [G_1(t-2) + G_1'(t-2)]$. Polygyny, $R(t)$, increases with the growth rate even with gender balance, $g=1$. Initial conditions do not affect the steady state growth rates.

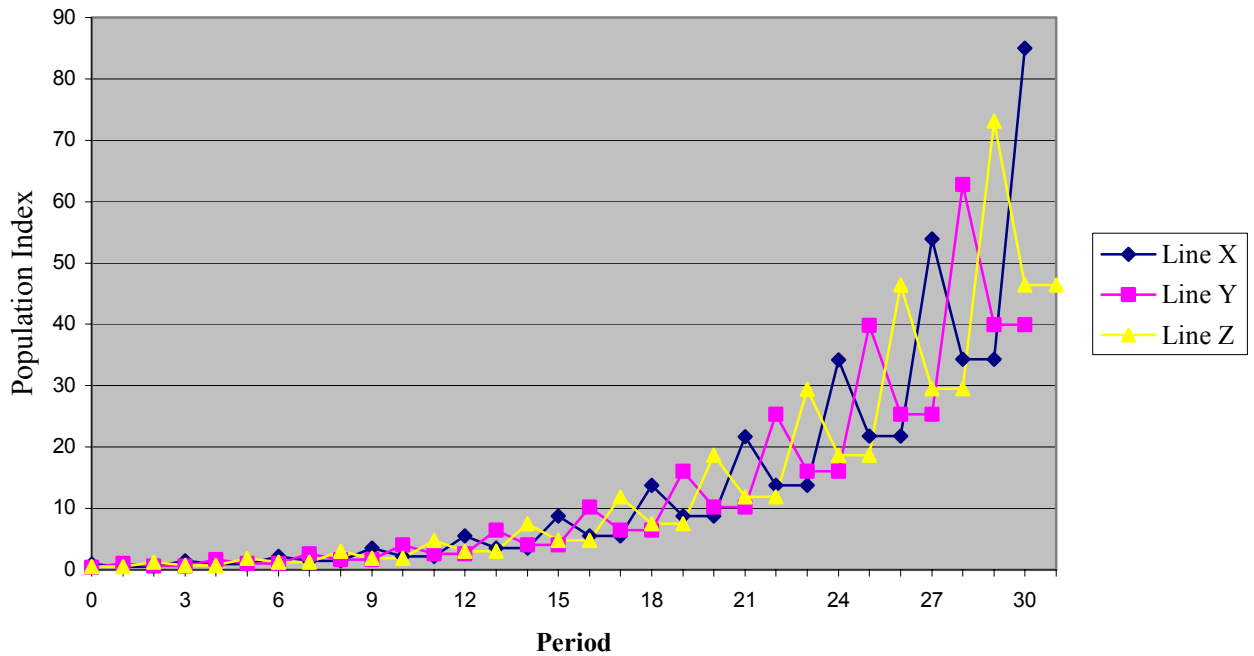
Maintaining symmetry across lines but allowing $p' > p$ yields very similar results. An example is given in Simulation 1. For comparison purposes, the simulation uses the same parameters (except for p_X) derived in the calibration analysis of Section 5.

²⁵ With full symmetry the dynamical system can be expressed by with two difference equations in $G_1(t+1)$ and $G_1'(t+1)$. In the zero population steady state, $G_1(t+1) = G_1(t)$ and $G_1'(t+1) = G_1'(t)$ for all t . Solving yields a condition for zero population growth, $pn^y + (1-p)n^o = 1$. Engineer and Kang (2005) prove that population grows if and only if $pn^y + (1-p)n^o > 1$.

Simulation 1: Symmetric age-set lines

$$p_X = p = .88, p' = .945, n_1^y = 1.286, n_2^y = n^o = .246$$

Populations of Women by Line



Though $n^o < 1$, population growth occurs because $n^y = 1.286$ is sufficiently large. The age-set lines are symmetric except for being in different phases. The steady state net average growth rate of the population is 16.36% per (14-year) period, and $R = 1.1455$. Increasing p or p' increases the growth rate and R .

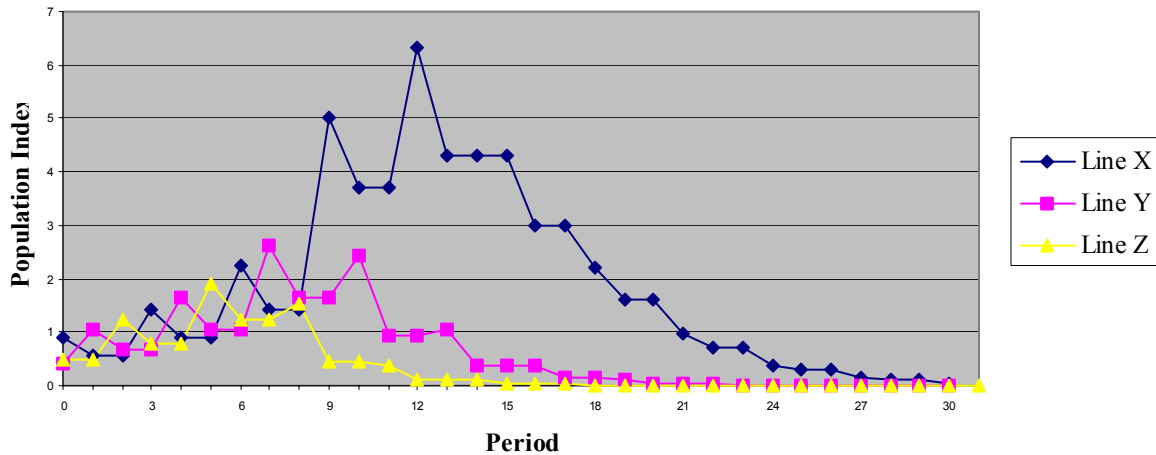
4.3 *Sepaade*

The effects of the institution of *Sepaade* can be examined by “shocking” the pre-existing symmetric steady state by lowering $p_X = p$ to $p_X < p$. The dynamic impact of this asymmetry is most clearly revealed by examining the extreme case $p_X = 0$, what we call a Complete *Sepaade* Shock. In this case, the *Teeria* population (line X) often comes to completely dominate the population, as illustrated in Simulation 2.

**Simulation 2: Complete *Sepaade*
Shock starting in period 5**

$$p_x = 0, p = .88, p' = .945, n_1^y = 1.286, n_2^y = n^o = .246$$

Populations of Women by Line



Apart from $p_x = 0$, Simulation 2 uses the same parameters as Simulation 1. After the shock in period 5, line X initially dramatically grows and the other lines (Y and Z) dramatically fall off. The other lines essentially disappear by period 30 and the *Teeria* line comprises the entire population. The steady state growth rate converges to $(n^o)^{1/3} - 1 = -37.34\%$. Below we show this is a general pattern.

The startling result that lines Y and Z disappear is due to line X becoming an absorbing state. Introducing Complete *Sepaade* at the beginning of period 5 results in all early-born daughters of line X fathers working in period 5 rather than marrying. They become women-marrying-old in period 6 to men from age-set 6. Thus, the shock results in these line X men enjoying extra wives at the expense of age set Z men. With Complete *Sepaade*, all early-born daughters from line X marry back into line X. In contrast, women marry out of the other lines into line X, leading to the increase in the *Teeria* population. Thus, there is a net drain of women into line X and the other lines retain women at a rate that is below replacement. In the steady state, all line X daughters are *Sepaade* and marry back into line X.

To identify the effects determining the steady state, first consider the special case where reproductive rates are at replacement levels, $n^y = n^o = 1$. For the reasons above, Complete *Sepaade* results in only line X existing in the steady state. The line X population doubles in size so that the average population is $2/3$ of the steady state level before the shock. This example reveals that *Sepaade* has the *negative level effect* of reducing the population. Instead of one group of females reproducing each period, as in the symmetric pre-shock steady state, there are twice as many females producing every third period. It is the combination of the delay in reproduction and the elimination of lines that yields the negative level effect.

Another channel by which *Sepaade* can reduce population is through a *negative growth effect* from young mothers rearing more children than old mothers, $n^y > n^o$.

Results: The steady state after the shock behaves as follows:

- (i) When $n_1^y = 1$, $n_2^y < 1$ and $n^y > n^o = 1$, only the *Teeria* age-set line (line X) survives. After the shock, the periodic steady state has no population growth; whereas, before the shock, population growth was positive.
- (ii) With $n_1^y \geq n_2^y \geq 1$ and $n^o = 1$, all age-set lines survive and grow at the same average rate. The *Teeria* line has the largest population and line Z the smallest.

The results in (i) show that *Sepaade* can reduce the net population growth rate from 1 to 0. The dramatic drop in growth is related to the complete *Sepaade* shock eliminating the other lines. More generally, with the other lines gone, the net average growth rate of the population is determined solely by n^o and is $(n^o)^{1/3} - 1$.

The results in (ii) indicate that when the rearing rates are sufficiently high, all lines survive in the steady state. Then the growth rate of the population is determined by all the rearing rates. The other lines survive because women marrying into those lines rear enough children for the lines to reproduce. This occurs despite *Sepaade* preventing

women from marrying outside of line X. Line Y is larger than line Z because it marries young women from line Z.

Historically, as we show in the next section, $n^y < 2$ and $n^o < 1$. Thus, the results in (i) are the most applicable to the Rendille. Of course, the analysis assumes Complete *Sepaade*. However, the evidence suggests that *Sepaade* rule has been incompletely applied, $0 < p_x < p$. With incomplete *Sepaade* the other lines do not disappear. Nevertheless, the Complete *Sepaade* results are indicative, as the steady state line proportions are continuous in p_x . We explore the incomplete *Sepaade* case with simulations in the next section.

5. Data and Calibration

Using standard demographic variables, the model can be calibrated to find the extent to which the *Sepaade* rule applies.

5.1 Demographic Data

Roth (1993, 1999) analyzes survey data on daughters of men from three adjacent age-sets that span the age-set lines.²⁶ These age sets correspond to age-set 10 from line Y initiated in 1908, age-set 11 in line Z initiated in 1922, and age-set 12 in line X (*Teeria*) initiated in 1936. The sample contains 101 *Teeria* daughters (*Sepaade*) and 107 *non-Teeria* daughters (*non-Sepaade*). Though Roth makes no claims for this being a representative sample, it is the only evidence we have of the cross-section of daughters by line. We use the data ratio of *non-Sepaade* to *Sepaade*, $107/101 = 1.06$, as a final touchstone in the calibration. The predominance of *Sepaade* in the sample lends credence to the idea that the *Sepaade* rule leads to the dominance of the *Teeria* line.

Roth (1999) presents strong direct evidence that the *Sepaade* rule reduces fertility and delays marriage. The female net reproductive rate for *Sepaade* is $NRR_x = .76$,

whereas for *non-Sepaade* it is much higher, $NRR = 1.39$. Roth's data on age at marriage is presented in 5-year intervals. Among *Sepaade*, 34% marry before they are 25 and 52% before they are 30. Among *non-Sepaade*, 78% marry before they are 25 and 92% before they are 30. For purposes of the model, we are interested in the proportion of women-marrying-young (28 years and younger). One issue is how to divide those in the 24-29 year age cohort. We use two benchmarks. Extrapolation, using the weights in the adjacent cohorts, yields the proportion of women-marrying-young among *Sepaade* as $P_X = .40$ and among *non-Sepaade* as $P = .89$. In contrast, the "strictest adherence" to rules consistent with the data gives $P_X = .34$ and $P = .92$. In the analysis below we concentrate on ranges: $0.34 \leq P_X \leq 0.4$ and $0.89 \leq P \leq 0.92$.

5.2 Ranges for the Model's Parameters

From the demographic data we can uncover ranges for the model parameters. First, consider the following decompositions:

$$\begin{aligned} NRR &= n^y P + n^o (1 - P), \\ NRR_X &= n^y P_X + n^o (1 - P_X). \end{aligned}$$

Here the average net reproductive rate is decomposed into the net reproductive rates of women-marrying-young, n^y , and women-marrying-old, n^o . These two equations can be solved for the unknowns n^y and n^o , given our data for NRR , NRR_X , P , and P_X . The following table explores how ranges of (P, P_X) impact (n^y, n^o) .²⁷

P	P_X	n^y	n^o
0.92	0.34	1.477	0.391
0.92	0.4	1.487	0.275
0.9	0.34	1.503	0.378
0.9	0.4	1.516	0.256

²⁶ Spencer (1973), Beaman (1981), provide results from smaller and less comprehensive surveys.

²⁷ The calculations are rounded to 3 decimal places. For extreme value $P_X = 0$, the net reproductive rate for women-marrying-old in line X is equal to the net reproductive rates for *Sepaade*, $n^o = NRR_X = .76$. This is the extreme upper bound for n^o . Conversely, when $P = 1$, $n^y = NRR = 1.39$. This is the lower bound for n^y .

The first row describes the “strictest adherence to *Sepaade*” case (largest ratio $P/P_X = .92/.34$). This case gives the largest n^o and smallest n^y , and hence we can bound the ratio $(n^o/n^y) \leq 0.391/1.477 = 0.265$. The delay in marriage reduces the net reproductive rate by about 75% or more.

Now consider the decomposition $n^y = n_1^y + n_2^y$ and ratio (n_1^y/n^y) . The restriction $n^o \geq n_2^y$ requires $n_1^y \geq n^y - n^o$. The “strictest adherence to *Sepaade*” case gives the smallest difference $n^y - n^o = 1.086$. Thus, $n_1^y \geq 1.086$, and we can bound the ratio $(n_1^y/n^y) \geq 1.086/1.477 = .735$. Women-marrying-young predominantly rear children that are born in the first period after marriage. If the restriction does not bind, $n^o > n_2^y$, then we get a higher lower bound on n_1^y/n^y . For the reason that a higher bound seem less plausible and for the reason that the binding restriction identifies parameter values, we assume $n^o = n_2^y$ in the following analysis.

The proportions P and P_X can be decomposed into early-born and late-born daughters who are women-marrying-young:

$$P = pw + p'(1 - w)$$

$$P_X = p_X w_X + p'(1 - w_X),$$

where w and w_X are the proportions of early-born daughters in the respective cohorts. It can be shown that $w_X \geq w \geq n_1^y/n^y \geq 1.086/1.477 = .735$.

First consider p_X , the proportion of *Sepaade* who are women-marrying-young. Since $p_X \leq p \leq p'$, it follows that $p_X \leq P_X \leq 0.4$, consistent with Assumption 6. Realistic values of p_X are likely smaller than this upper bound. For example, if $p' = .9$ and $w_X = .82$, then late-born *Sepaade* who are women-marrying-young account for $(.9)(1-.82) = .162$ of women. If the total women-marrying-young is $P_X = .4$, it follows that $p_X = .290$. The value of p_X is increasing in P_X and w_X , and decreasing in p' . Thus, using values $P_X = .34$,

$p'=1$ and $w_X = 0.735$, we can establish a lower bound, $p_X > 0.10$. The data is inconsistent with Complete *Sepaade*.

Similarly, consider the lower bound for p . The value of p is increasing in P and w , and decreasing in p' . If $P = .89$, $p'=1$ and $w = .7353$, then $p = .85$. This is well within the lower bound of $p \geq .8$ specified in Assumption 6. In contrast, the highest value of p corresponds to $p = p' = P = .92$.

5.3 Calibration

The marriage timing parameter p , p_X and p' depend on w and w_X , the proportions of daughters that are early born. In turn, w and w_X are generated from the simulation and depend on all the model parameters. Since our data does not correspond to the steady state, w and w_X also depend on the initial conditions. As described above, *Sepaade* is analyzed as a shock from the symmetric steady state. The following table presents calibration results for various (P, P_X) combinations under the assumption that $n_2^y = n^o$.

P	P_X	n^y	n^o	p	p'	p_X	NSR
0.92	0.34	1.477	0.391	.92	.92	.189	.795
				.892	1	.168	.774
0.92	0.4	1.487	0.275	.92	.92	.302	1.008
0.9	0.34	1.503	0.378	.9	.9	.315	.825
0.9	0.4	1.516	0.256	.9	.9	.315	1.051
				.88	1	.30	1.023
.89	.39	1.529	0.269	.89	.89	.303	1.030
.89	.40	1.531	0.246	.89	.89	.322	1.075
				.87	1	.305	1.048
				.88	.945	.3125	1.060
.89	.41	1.534	0.222	.872	1	.325	1.095

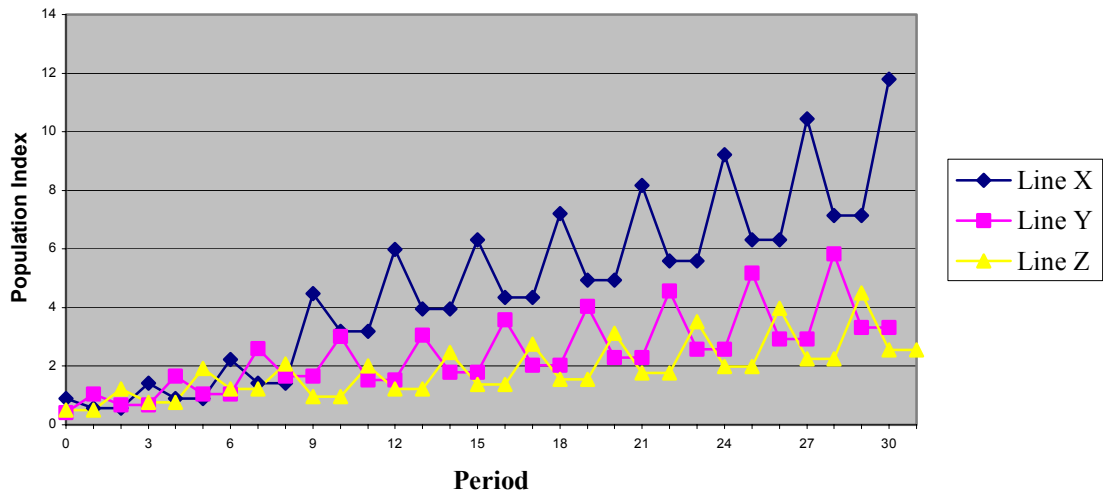
As described before, each (P, P_X) pair uniquely yields a (n^y, n^o) pair. Given (n^y, n^o) , the model is calibrated by finding values of p, p' and p_X that generate (P, P_X) output over the sample (periods 10-12) consistent with the inputted (n^y, n^o) . Given (n^y, n^o) , for each p' there is a unique solution for p and p_X . The polar cases $p'=p$ and $p'=1$ are reported. The lowerbound for $p', p'=p=P$, corresponds to the upperbound for $p, p=P$. The values of p' and p move inversely, so that $p'=1$ corresponds to p being at its lowerbound. Similarly, all the simulations display an inverse relationship between p' and p_X and between p' and *NSR* (the *Non-Sepaade* to *Sepaade* Ratio).

The first four (P, P_X) pairs in the table are the cases discussed in the data Section 5.1. None of these entries give calibrations with *NSR* = 1.06, the ratio of *Non-Sepaade* to *Sepaade* found in the data. However, the “base case” $(P, P_X) = (.9, .4)$ comes surprisingly close, and $(P, P_X) = (.89, .40)$ yields ratios $1.048 \leq NSR \leq 1.075$. With $(P, P_X) = (.89, .40)$, the unique calibration that exactly matches *NSR* = 1.060 has $p = .88, p' = .945$, and $p_X = .3125$. As $p_X = .3125$, the calibration indicate a substantial level of non-compliance with the *Sepaade* rule. Simulation 3 illustrates the dynamics.

**Simulation 3: Calibration to *NSR* = 1.06
Shock starting in period 5**

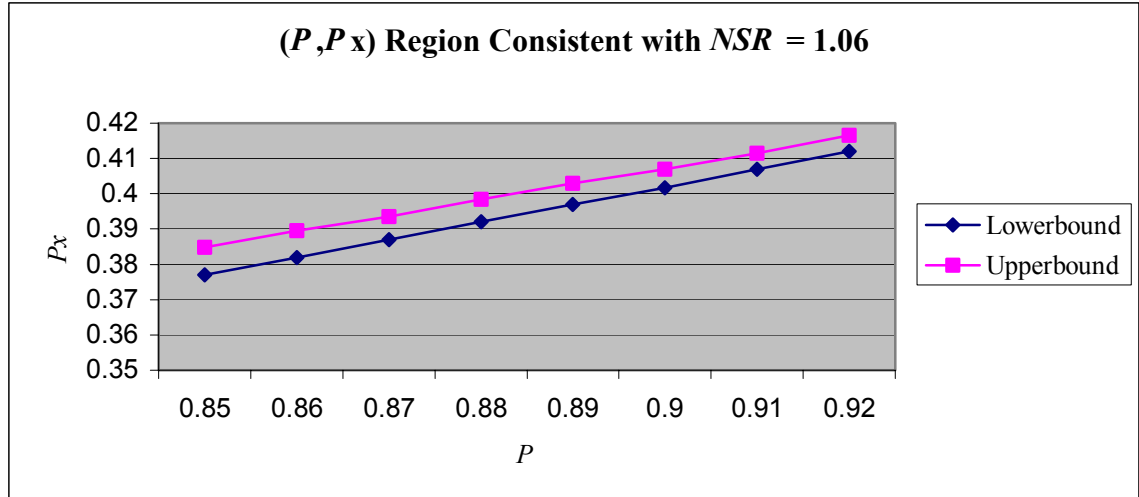
$$p_X = .3125, p = .88, p' = .945, n_1^y = 1.286, n_2^y = n^o = .246$$

Populations of Women by Line



After the shock the total population continues to grow and converges to a (average) growth rate of 4.2% per period. This is in stark contrast to the Complete *Sepaade* case (Simulation 2 which has the same parameters except for $p_X = 0$), where the total population starts to fall by period 13 and converges to a very negative (average) growth rate of -35.0% . Nevertheless, the incomplete adherence to the *Sepaade* rule does reduce the growth rate very substantially from the 16.4% that would arise in the absence of the rule (see Simulation 1). Also, in contrast to the Complete *Sepaade* case, lines Y and Z exist in the steady state where the ratio of Non-*Sepaade* to *Sepaade* daughters in the steady state is 1.078. Thus, in the transition, this ratio overshoots its steady-state level.

Of course, this is not the only possible exact calibration. The follow chart shows that by varying P and P_X together there is a substantial region of plausible (P, P_X) parameters for which we can generate calibrations with $NSR = 1.06$.

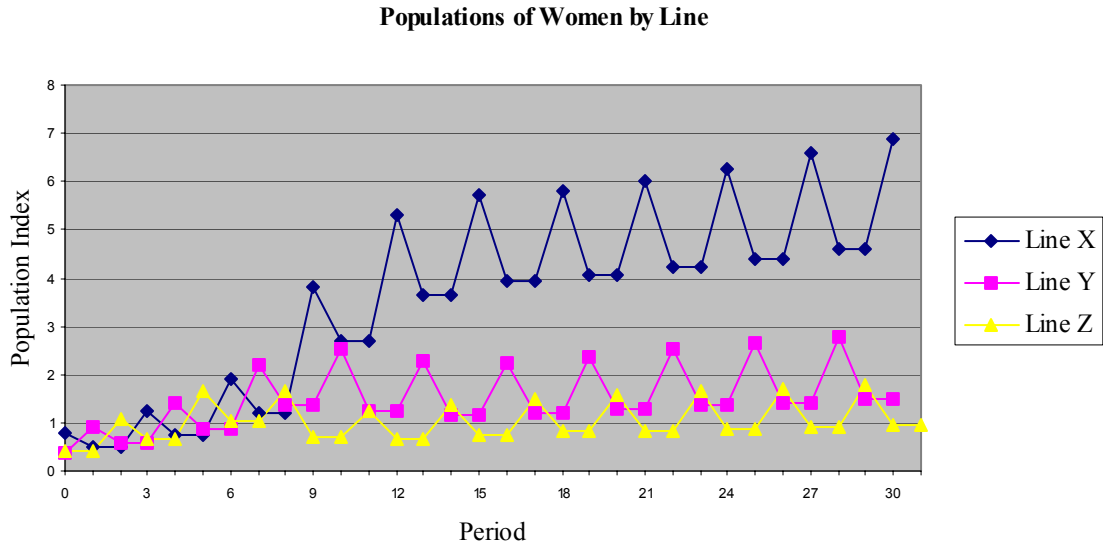


The lowerbound in the chart corresponds to simulations with $p = p' = P$; whereas, the upperbound has $p < P < p' = 1$. Overall these parameters range $p \in [.82, .92]$ and $p' \in [.85, 1]$. In contrast, the calibrations provide a much narrower range for $p_x \in [.293, .327]$. Thus, for calibrations with $NSR=1.06$, we can conclude that slightly less than a third of early-born *Sepaade* marry young in violation of the rule. Generically, all of these calibrations have similar features to Simulation 3: there is positive population growth and lines Y and Z produce about half the total daughters.

Of course, these conclusions are based on fitting the model to $NSR = 1.06$. But as mentioned in the data section, this data ratio is suspect (as representative of a random cross-section). The natural question is: How robust are the conclusions to all plausible (P, P_x) pairs? In particular, are there any (P, P_x) pairs that give negative growth and highly skewed population proportions? Consider the first entries in the table, $(P, P_x) = (.92, .34)$. This corresponds to the “strictest interpretation of *Sepaade*”. Not surprisingly, these entries have the highest values of p and p' and lowest values for p_x . The entry with the smallest p_x has the lowest NSR value and a growth path with the smallest population and the lowest growth rates. The growth path is illustrated in Simulation 4.

**Simulation 4: Strictest Adherence to *Sepaade*
Shock starting in period 5**

$$p_X = .168, p = .892, p' = 1, n_1^y = 1.477, n_2^y = n^o = 0.391$$



The growth path is similar to Simulation 3 but with less growth converging to a steady state growth rate of 1.5%. Thus, even with the “strictest adherence to *Sepaade*, growth is positive and lines Y and Z exist in the steady state (the ratio of Non-*Sepaade* to *Sepaade* in the steady state is 0.793, also indicating overshooting). This contrasts with a steady state growth rate of -23.9% that results when $p_X = 0$ and a growth rate of 15.5% when $p_X = p$, *ceteris paribus*.

5.4 Discussion

The candidate values of (P, P_X) consistent with the data, yield calibrations with quite different growth rates and proportions. Nevertheless, several strong conclusions emerge. First, the *Sepaade* rule is incompletely adhered to: it holds back the majority but not all early-born *Sepaade* daughters from marrying young, $0.15 < p_X < 1/3$. Second, though the *Sepaade* rule is incompletely adhered to, it substantially reduces the growth

rate of the population from high rates (15.5 % and 16.4%) to low rates (1.5% and 4.2%). Third, the *Sepaade* rule results in the *Teeria* line being about as populous as the other two lines together (*NSR* about 1).

Other evidence supports these conclusions. Beaman (1981) survey of *Teeria* age-set 12 finds that approximately one quarter of all eligible women do not follow the *Sepaade* role, so $p_X = .24$. Though there is no good historical data, it would appear that the Rendille population has been growing fairly slowly. In 1990 they numbered at about 30,000.²⁸ Rendille elders report that the *Teeria* is by far the largest and most powerful line, as large and as powerful as the other two lines combined.²⁹

The analysis also reveals other interesting features. After the shock the total population displays a marked three-period population cycle, which persists in the steady state. This cycle is the same length as a *fahan* (a rotation through the age set lines). The steady state of our model does not produce the six-period cycles of boom and bust believed to exist among the Rendille and termed *fahano*.³⁰ If this cycle is to show up in the demographics it must come from another source (e.g. war, disease, ecology). Indeed, the progression of *fahano* is usually associated with alternating periods of peace then war. The three period demographic cycles would provide the natural building block of a six-period cycle.

6. The Political Economy of *Sepaade*

The origin, role, and the recent end of the *Sepaade* tradition are detailed in Roth (2001). An emic view is that *Sepaade* was an institutional response to prolonged heavy warfare with Somali neighbours in the early 19th century. Young women of marrying age

²⁸ By lowering the steady state population growth, *Sepaade* also lowers the steady state polygyny ratio on average. Our analysis probably underestimates the extent of polygyny because we do not allow for the emigration of sons who receive no inheritance because of primogeniture. Nevertheless, the historical observation is that most men had one wife and very few had more than two wives.

²⁹ Information revealed in interviews taped by Merwan Engineer in 2001.

³⁰ Beaman's rule 14 is "Fahano influence history for good or ill in alternating periods of 42 years for a cycle of 84 years. Thus, every age-set is associated with a period of historical influence characterized by either peace or war which alternates every 42 years as predictably as the seasons."

were recruited to take care of the camels when the warriors engaged the enemy. This precluded them from marrying. It also made the nomadic Rendille more mobile, not having to carry young children when relocating. According to this account, the institution results from a cultural group selection arising as an emergency response to a crisis. While the institution may have been beneficial to the community, it seriously constrained women's fertility and thus is considered disadvantageous to *Sepaade*.

The institution of *Sepaade* was introduced in years 1825-1839, period 5 in the model. The impact of the institution was to prevent women of line X from marrying young to men of line Z in 1836. Instead, these women marry when old to men from line X in 1851, period 6 in the model. The, *Sepaade* shock results in line X men enjoying extra wives at the expense of line Z men. This shows up in the simulations with a dramatic increase in polygyny in line X in period 6. There is a dramatic decrease in polygyny in line Z in period 5, but if many line Z men had died in battle the actual polygyny ratio may not have fallen.³¹

Sepaade has major implications for work and wealth. *Sepaade* keeps women from line X doing hard work for their fathers for an extra period rather than marrying when young. Thus, the institution generates an immediate and ongoing increase in labour from women of line X. Furthermore, *Sepaade* keeps wealth within the *Teeria* line -- the bride wealth of four camels is paid to a *Teeria* father. (Men in line X eventually inherit their wealth from their *Teeria* fathers.) Thus, introducing *Sepaade* unambiguously benefits men of the *Teeria* age set line. Conversely, it is disadvantageous to men in the other lines.

Why did the *Sepaade* institution persist, well after the external threat had passed? This is puzzling for at least two reasons (ignoring population regulation). First, the dynamics reveal that the composition effect from *Sepaade* makes the lineages unbalanced. Thus, the Rendille are increasing at a disadvantage for wars breaking out in

periods in which line Y and Z men are warriors. The second reason is that Y and Z men were at least initially in the majority and their self-interest would have been to abolish the institution.

In fact, there was at least one earlier attempt to abolish the institution. Engineer in interviews with Rendille elders in 2001 was told that the Rendille had convened a council to consider the abolition of *Sepaade* in 1966 (period 15). In that council, all elders but those from two senior *Teeria* families were for abolition. Nevertheless, they were able to block abolition. In 1998 (period 17) they relented and the institution was abolished. The interviews revealed that Rendille collective decision-making normally requires a very high plurality. Further, the *Teeria* elders, as powerful members of the “first-born” lineage, have extra clout in decision-making. This would explain why *Sepaade* persisted. But the dynamics would suggest that the *Teeria* with time would be in an even stronger position. So a puzzle remains: if the *Teeria* were able to block abolition in the past they should be to block it in 1998.

An answer to this puzzle was provided in an interviews, including akey elder from the most powerful *Teeria* family that blocked the move in 1966. The elders confirmed that the *Teeria* stood to gain substantially from *Sepaade* and for that reason had blocked the change in 1966. However, all the *Teeria* had agreed to abolition in 1998 because the early-born *Sepaade* daughters were already starting to escape in anticipation of being forced to hard work (instead of being allowed to marry in 2004). Apparently, *Sepaade* escaping to neighbouring tribes, including traditional enemies, had occurred in 1966. However, the exodus was forecast to be worse this time, perhaps because of the ability to escape to the cities. Faced with the inability to keep their young daughters from running away the *Teeria* elders agreed to the abolition. Thus, it would appear that a change in the participation constraints facing *Sepaade* that explain abolition.

7. Conclusion

³¹ The impact is somewhat muted if men of age set line Z can marry late in period 6, at the end of the fourth period of their lives. Sons unless they climb join line X. Daughters on the other hand are married to line Z.

In this paper, we draw on anthropologists' reports to model the social rules of a particular age-group society, the Rendille of Northern Kenya. We show that the social rules imply an almost structurally exact overlapping generations (OLG) model. By “almost structurally exact”, we mean that the transitions by which individuals transit the lifecycle almost exactly correspond to the beginning of the 14-year period intervals.

We develop a parsimonious model, where individuals are potentially six-period lived, that captures the essential heterogeneity in the timing of marriage and birth. The Rendille provide a particularly good case study because their age-group system has lineages integrated with their age set system. This allows us to track lineages and examine the intergenerational political economy behind the social rules. As far as we are aware, there are no applied OLG models that are structurally exact, capture marriage and birth timing heterogeneity, or track lineages.

Another novel feature of the paper is that the OLG model can be calibrated out of the steady state using standard (time and group aggregated) cross-section demographic data. The calibration allows us to derive parameters that correspond to specific individual lifecycle transitions: period fertility rates conditional on age at marriage and marriage timing probabilities conditional on birth order. The parameters can also be derived according to age groups.

In our application, we examine the marriage timing parameters by lineage groups and the asymmetry implied by the *Sepaade* rule. Strict application of the *Sepaade* rule requires that no early-born daughters of a particular lineage (the *Teeria*, line X) marry early, $p_X = 0$, whereas it is the usual practice in other lineages, $p > 0.8$. Our calibration analysis finds $0.15 < p_X < 1/3$ and $p > 0.85$ implying that the rule is incompletely applied. Nevertheless, the implied delay in marriage dramatically lowers the path of the population leading to almost zero population growth.

Changing this initial specification does not change the steady state impact of *Sepaade*.

Analysis of the dynamic model identifies a negative level effect, a negative growth effect, and a composition effect. The composition effect shows that the delay in marriage favours line X and this line quickly becomes as populous as the other two lines combined. These effects are much more pronounced if the rule is strictly followed. Then the population falls after 7 periods and converges to rapid negative growth. The composition effect involves line X quickly comprising the entire population as the other lines disappear.

Our results are consistent with the view in the anthropology literature that the *Sepaade* rule regulates constant population³² However, our analysis reveals that it is the incomplete application of the rule that maintains constant population. This raises questions about how the rule is regulated and questions about the intergenerational political economy supporting the rule. The *Sepaade* institution was shown not only to favour the *Teeria* by increasing their numbers but also by increasing their wealth. Both factors added to their political clout in blocking attempts to abolish the institution by the other lines.

The model throws up the possibility that the institution was abolished in 1998 because it was leading to a society which was too lopsided and hence vulnerable to attack. However, interviews with Rendille elders suggest the more plausible reason that it was the increased ability of *Sepaade* daughters to escape to the neighbouring tribes and the cities that explains the recent abolition. Thus, a change in the participation constraints of the *Sepaade* appears to be a decisive factor in limiting the power of the *Teeria* men.

Whereas we have explored the rich dynamic implications of the structural rules of Rendille society, our analysis falls substantially short of a full account. The gold standard in economics is to specify an environment and preferences and then examine the

³² Roth (1993) reviews this literature, which argues that *Sepaade* prevents overpopulation and thereby helps achieve homeostasis with the environment.

equilibrium behavioral choices for consumption, production and reproduction.³³ In future work we hope to acquire more data and knowledge to model important aspects of individual choice in a general equilibrium framework.³⁴ Nevertheless, we believe that the structural analysis shows that the OLG model is a powerful tool for investigating the nature and dynamics of actual societies. We hope that this exercise helps points the way to modifying the OLG model as an applied framework for societies that less exactly fit the standard model.

REFERENCES

Auerbach A. and L. Kotlikoff (1987), *Dynamic Fiscal Policy*, Cambridge University Press: New York.

Beaman, A.W. (1981), "The Rendille Age-Set System in Ethnographic Context: adaptation and integration in a nomadic society", unpublished Ph.D. dissertation, Department of Anthropology, Boston University.

Engineer, M. H. and D. Bernhardt (1992), "Endogenous Transfer Institutions in Overlapping Generations", *Journal of Monetary Economics*, 29, 445-74.

Engineer, M. H., Esteban, J. and Sakovics (1997), "Costly Transfer Institutions and the Core in Overlapping Generations", *Journal of Economic Behavior and Organization*, 32, 287-300.

_____ and M. Kang (2003), "An Overlapping Generation-Group Model of the Rendille of Northern Kenya", mimeo, Department of Economics, University of Victoria.

³³ A full account would address the more fundamental questions of why social rules arise. Our analysis reveals that certain rules are more likely to be stable than others. In the economics literature, Engineer and Bernhardt (1992) look at the incentive compatibility conditions between two-period lived generations and Engineer, Esteban and Sakovics (1997) examine the core of a simple OLG to examine the stability of age-based social rules.

³⁴ Roth's (1999) work on marriage choice points to specifying preferences over cattle and camels for wealth and also the probability of a surviving male heir. Production mainly involves camels, and there is surprisingly detailed information available about camels in the region. Perhaps not surprisingly, camels appear to have a lifecycle that coincides with the 14 year age-set period. Droughts and human cycles may induce cycles in the environment that have their own "deep-ecology" dynamic. From this perspective, the institutions of age-sets and *Sepaade* may be viewed as intriguing ecological control mechanisms.

Engineer, E. Roth and L. Welling (2004), "Modeling Social Rules and Experimental History: The Impact of the *Sepaade* Tradition on the Rendille", mimeo Department of Economics, University of Victoria.

Engineer, M. H., and L. Welling (2004), "Overlapping Generations Models of Graded Age-Set Societies", *Journal of Institutional and Theoretical Economics*, 160(3), 454-476.

Gokhale, J. L., J. Kotlikoff, J. Sefton, and M. Weale (2001), "Simulating the Transmission of Wealth Inequality via Bequests", *Journal of Public Economics*, 79, 93-128.

Greenwood, J., N. Guner, and J. Knowles (2003), "More on Marriage, Fertility, and the Distribution of Income." *International Economic Review*, 44, 827-662.

Hammel, E. A. (1979), "Experimental History", *Journal of Anthropological Research*, 35, 274-291.

Huggett, M. (1996), "Wealth Distribution in Life-Cycle Economics", *Journal of Monetary Economics*, 38, 469-494.

Legesse, A. (1973), *Gada: Three Approaches to the Study of African Society*, Free Press.

Roth, E. (1993), "A Re-examination of Rendille Population Regulation", *American Anthropologist*, 95:597-611.

_____ (1999), "Proximate and Distal Variables in the Demography of Rendille Pastoralists", *Human Ecology*, 27. 4: 517-36.

_____ (2001), "Demise of the Sepaade Tradition: Cultural and Biological Explanations", *American Anthropologist*, 103.4: 1014-1024.

Spencer, P. (1997), *The Pastoral Continuum*, Oxford University Press.

Stewart, F.H (1977), *Fundamentals of Age-Group Systems*, Academic Press.